

occurs as in a torsion test-pieces. By only considering a material, which in the course of a torsion test shows an upper yield shear stress τ_y and a lower yield shear stress τ'_y , giving rise to a yield without hardening, following equation $\sigma_t - \sigma_r = 2\tau'_y$ may be written and will be applicable everywhere in the plastic zone, provided that the overstraining is not too excessive which occurs, when k is not too great. With these restrictions, the equilibrium of the forces obeys eq. (2), which is applicable in the elastic as well as in the plastic zone and takes actually following form

$$\sigma_t - \sigma_r = 2\tau'_y = l \frac{d\sigma_r}{dl} \quad \text{or} \quad \int_l^m d\sigma_r = \tau'_y \int_l^m d \log l^2.$$

The right hand side of the integrated equation is equal to $\tau'_y \log (m^2/l^2)$; the left hand side of said equation is equal to $(\sigma_{r=m r_1} - \sigma_r)$; $\sigma_{r=m r_1}$ can be extracted from eq. (16) by equating l to m , because the stress σ_r is certainly continuous throughout the wall; eq. (21) of table 2 is based on these indications; following equations of table 2 are found as follows : eq. (22) derives from eq. (21) and following relation : $\sigma_t = \sigma_r + 2\tau'_y$; eq. (24) derives from eqs. (21) and (22) and from following equation :

$\sigma_z = \frac{1}{2}(\sigma_r + \sigma_t)$ eq. (23) is mentioned as a reminder. It will be noted, that the radial displacement u in the plastic zone is not mentioned in table 2. This is accounted for by the fact, that the relation between the stresses and the resulting plastic strains depends upon the loading history.

It must be now ascertained, that the axial equilibrium of the forces is satisfied by Cook's assumption by means of which σ_z can be calculated in the elastic as well as in the plastic zone. This equilibrium is expressed by following relation

$$2\pi \int_1^m \sigma_z (\text{plast}) (lr_1) d(lr_1) + 2\pi \int_m^k \sigma_z (\text{elast}) (lr_1) d(lr_1) = \pi r_1^2 p_1$$

or simpler by following relation

$$\int_1^m \sigma_z (\text{plast}) dl^2 + \int_m^k \sigma_z (\text{elast}) dl^2 = p_1.$$

By making use of eqs. (19) and (24), one can verify, that the left hand side of the above equation reduces to : $(1 - m^2/k^2) \tau_y + (\log m^2) \tau'_y$, which is precisely the value of the internal pressure, which value is obtained by putting down in eq. (21) $\sigma_{r=r_1} = -p_1$ and $l = 1$.

Cook's hypothesis is thus acceptable. One can even say, that this theory sufficiently explains the facts, if we consider, that it is a simplified one. These

views are justified by theoretical and experimental considerations. COOK [1934] and CROSSLAND, JÖRGENSEN and BONES [1958] made use of Cook's hypothesis for interpreting their experiments and ALLEN and SOPWITH [1951] have demonstrated, that this hypothesis fits in well with their more elaborated theoretical solution. Table 2 improves a theory of the "autofrettage" expounded by DEFFET and GELBGRAS [1953]. It is also to be noted, that table 2 is applicable to a Maraging steel, when τ'_y is made equal to τ_y .

An m value being selected, one can find by means of the equations of table 2, how stresses are distributed in the elastic and plastic zones, when l varies. Then, by changing the m value, one can see how this stress distribution modifies, until the overstraining is completed ($m = k$). As the outer radius displacement u_2 is a dependent variable of the pressure, its variation can be found as follows: the displacement is extracted from eq. (20), where one puts down $l = k$ and pressure p_1 is extracted from eq. (21) where one puts down $\sigma_r = -p_1$ and $l = 1$; as m is a quantity, which may vary from $m = l$ to $m = k$, a diagram can be plotted by means of the corresponding u_2 and p_1 values. CROSSLAND, JÖRGENSEN and BONES [1958] used such a diagram as a reference curve, when they analyzed the results of tests, they carried out on a low-alloy steel. The results and the curve, show a very satisfactory concordance, although the dispersion of τ_y and τ'_y were not negligible.

The same authors have calculated the pressure p_{1y} by taking eq. (15) into consideration and using a corrected value of τ_y they have also calculated the "collapse" pressure p_{1c} after eq. (22) by putting down: $p_{1c} = -\sigma_r$, $m = k$ and $l = 1$

$$p_{1c} = \tau'_y \log k^2. \quad (25)$$

The results they have obtained by testing cylinders, made of a 0.15% carbon steel and of which the k ratio varied between $k = 1.5$ and $k = 6$ are in excellent agreement with eq. (25).

Although the simplified theory has been largely confirmed by tests, carried out to this purpose, such a theory cannot be applied to self-hooping a cylinder, without taking some precautions. In fact, when the pressure goes down, the cylinder shows residual stresses. The normal direction of the shear stress in the plastic zone is reversed and it may be feared, that as a consequence of the Bauschinger effect, a plastic deformation in the opposite direction prematurely appears. This effect can be detected on the hysteresis loop of the pressure vs. deformation curve pertaining to a cylinder submitted to pressures going up and down. When a cylinder at rest shows a plastic reversed deformation and when the pressure, to which it is submitted, goes